ECE317 : Feedback and Control

Lecture :
Design using Bode plots, compensation

Dr. Richard Tymerski
Dept. of Electrical and Computer Engineering
Portland State University

Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Block Diagram
- ✓ Linearization
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ example system

Analysis

- ✓ Stability
  - ✓ Pole locations
  - ✓ Routh-Hurwitz
- ✓ Time response
  - ✓ Transient
  - Steady state (error)
- ✓ Frequency response
  - ✓ Bode plot

Design

- Design specs
- Frequency domain
- Bode plot
- Compensation
- Design examples

Matlab & PECS simulations & laboratories

Notes on Bode plot (review)

- Advantages
  - Without computer, Bode plot can be sketched easily by using straight-line approximations.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple.

Compensators

1) Proportional (P) compensator:

$$G_c(s) = k_p$$

2) Dominant pole (I, integrator) compensator:

$$G_c(s) = \frac{\omega_I}{s}$$

3) Dominant pole with zero (PI, proportional plus integrator)

$$G_c(s) = \frac{\omega_I}{s}\left(1 + \frac{s}{\omega_z}\right)$$

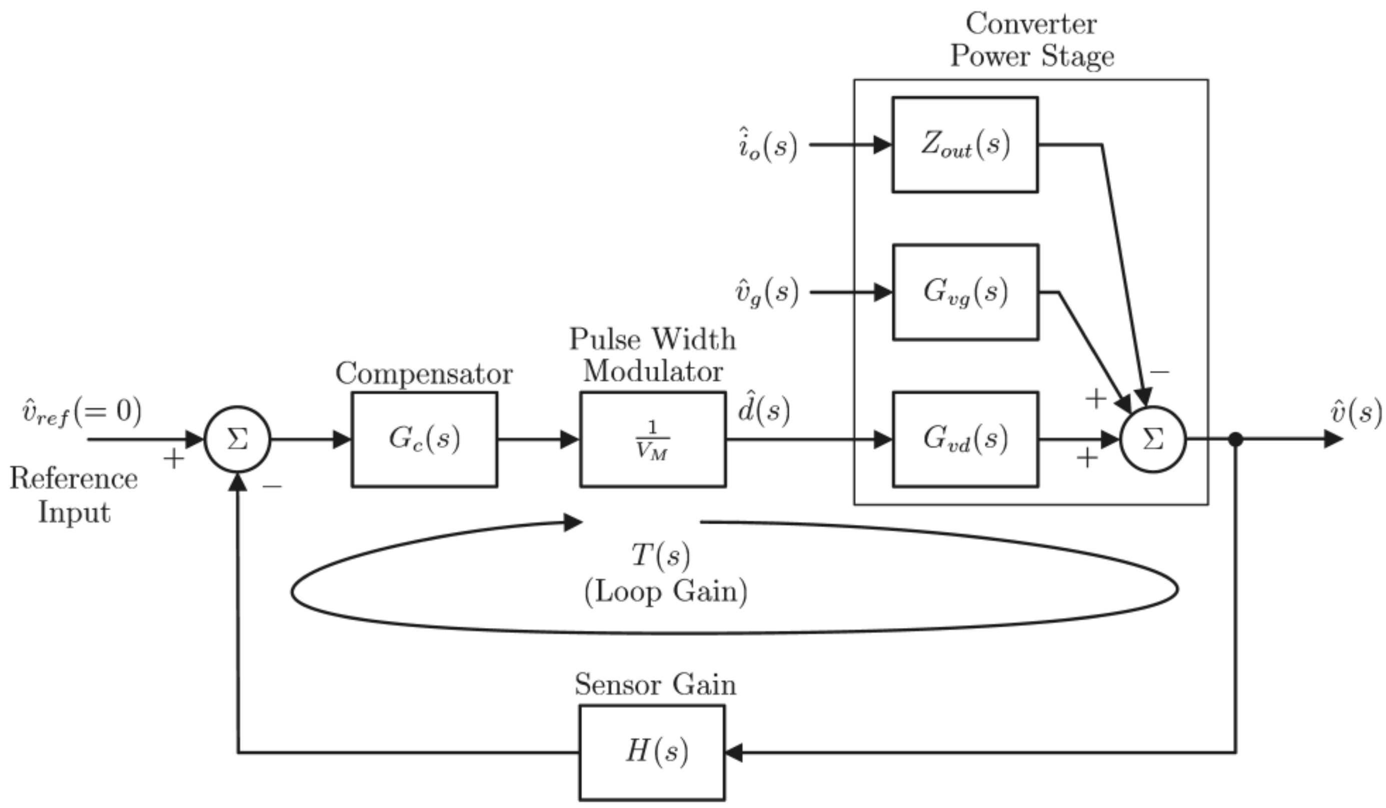
4) Lead compensator:

$$G_c(s) = G_{c_o}\frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \qquad \omega_z < \omega_p$$

5) Lead with integrator and zero compensator

$$G_c(s) = \frac{\omega_I\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)}{s\left(1 + \frac{s}{\omega_p}\right)}$$

Application to the lab:



$$T(s) = \frac{1}{V_M} \cdot G_c(s) \cdot G_{vd}(s) \cdot H(s)$$

Uncompensated System

Loop Gain, T(s):

$$T(s) = \frac{1}{V_M} \cdot G_c(s) \cdot G_{vd}(s) \cdot H(s)$$

where

$$G_{vd}(s) = \frac{V_g}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$Q = \frac{\sqrt{LC}}{r_L C + \frac{L}{R}}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$G_c(s) = 1$$

Uncompensated System

Loop Gain, T(s):

$$T(s) = \frac{T_o}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

where

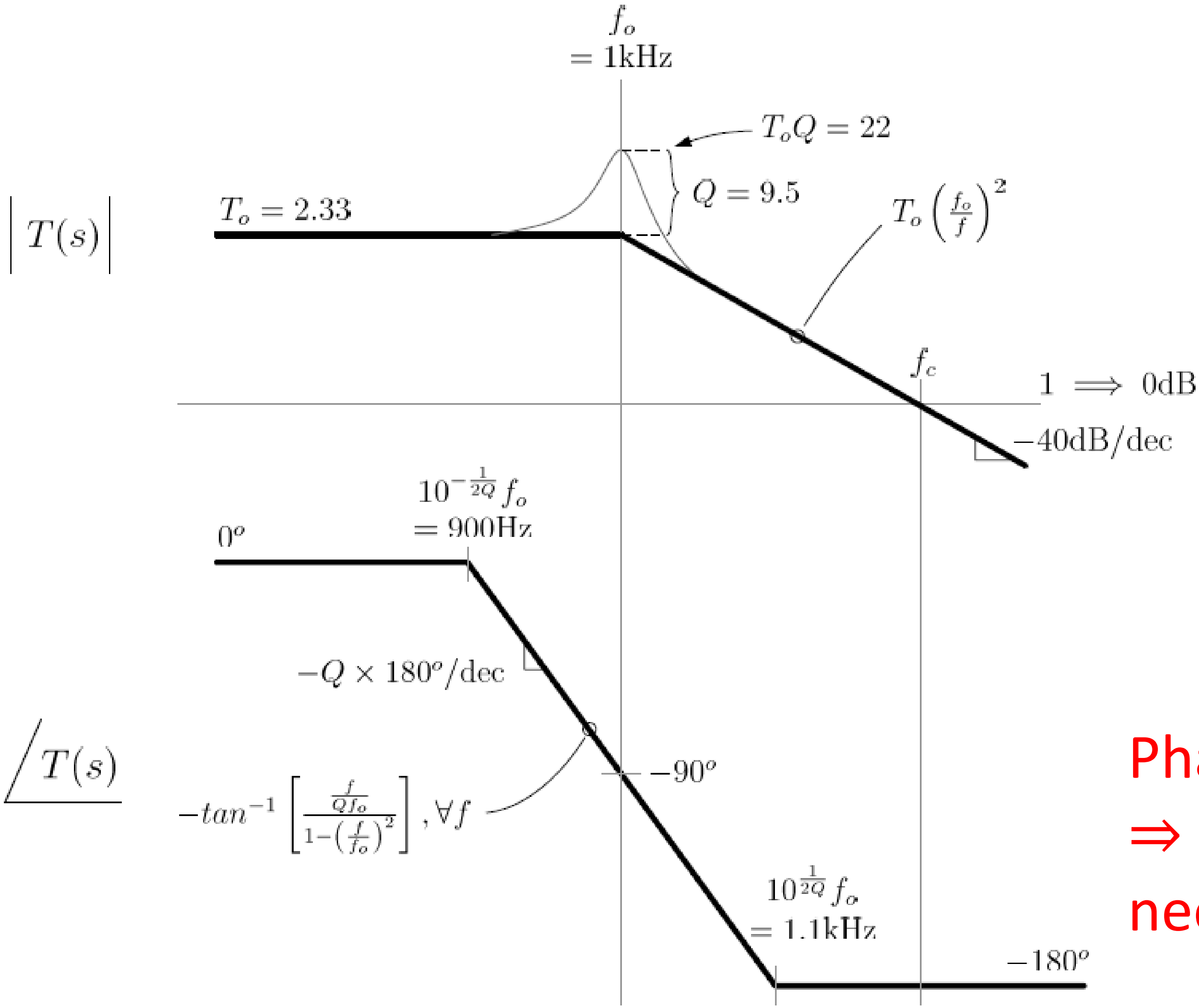
$$T_o = \frac{V_g H(0)}{V_m}$$

Example used here:

$$T_o = 2.33, \quad Q = 9.5, \quad \omega_0 \implies f_0 = 1kHz$$

Uncompensated System

Asymptotic Bode plot:



$$T_o\left(\frac{f_o}{f_c}\right)^2 = 1$$

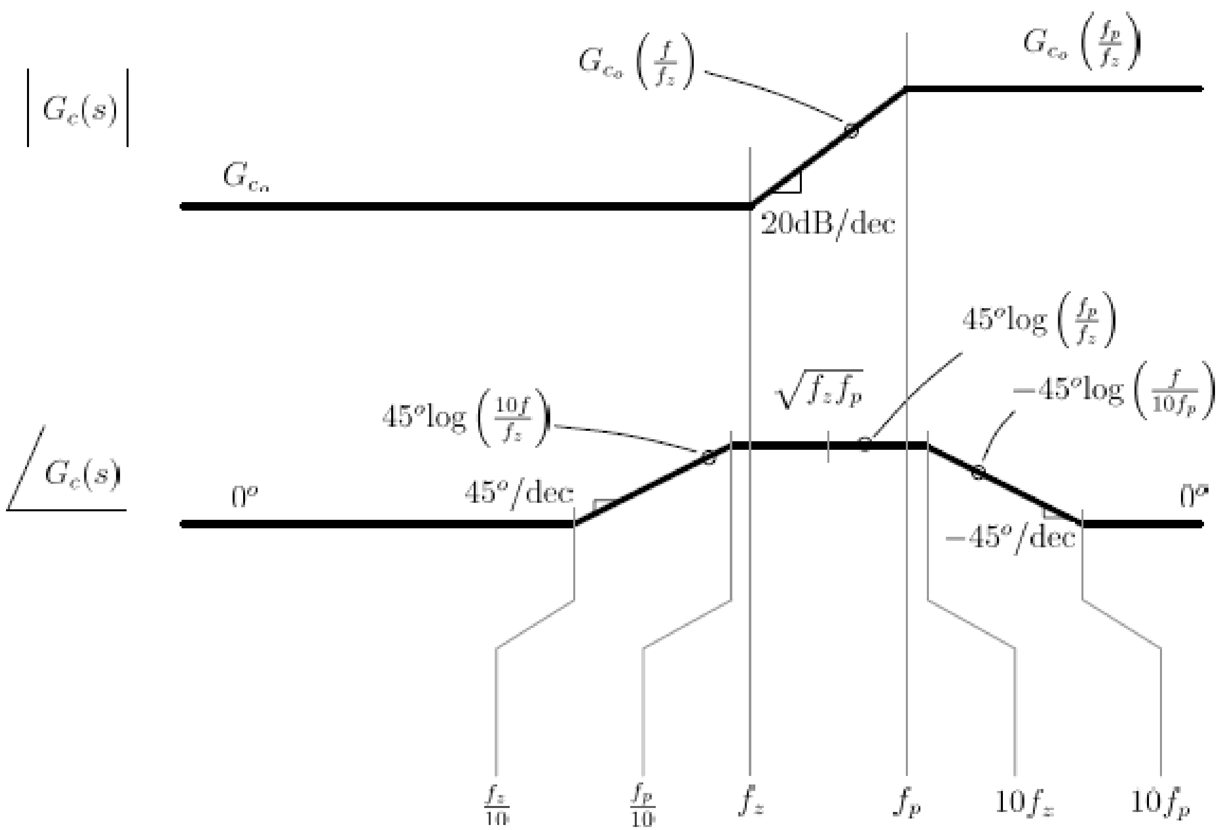
$$\implies f_c = 1.5kHz$$

Phase margin = 0°
⇒ compensator is needed

Lead Compensation

$$G_c(s) = G_{c_o} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \quad \omega_z < \omega_p$$

Asymptotic Bode plot:



Lead Compensation

- The basic idea of using a lead compensator is to provide a phase boost at the unity gain crossover frequency
- Will extend bandwidth (i.e. unity gain crossover frequency) while also providing phase boost
- Maximum phase boost occurs at: $f = \sqrt{f_z f_p}$
- Set *f* to the new crossover frequency: $f_c = \sqrt{f_z f_p}$

Lead Compensation

Lead compensator transfer function:

$$G_c(s) = G_{c_o} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \quad \omega_z < \omega_p$$

Lead Compensated Loop Gain:

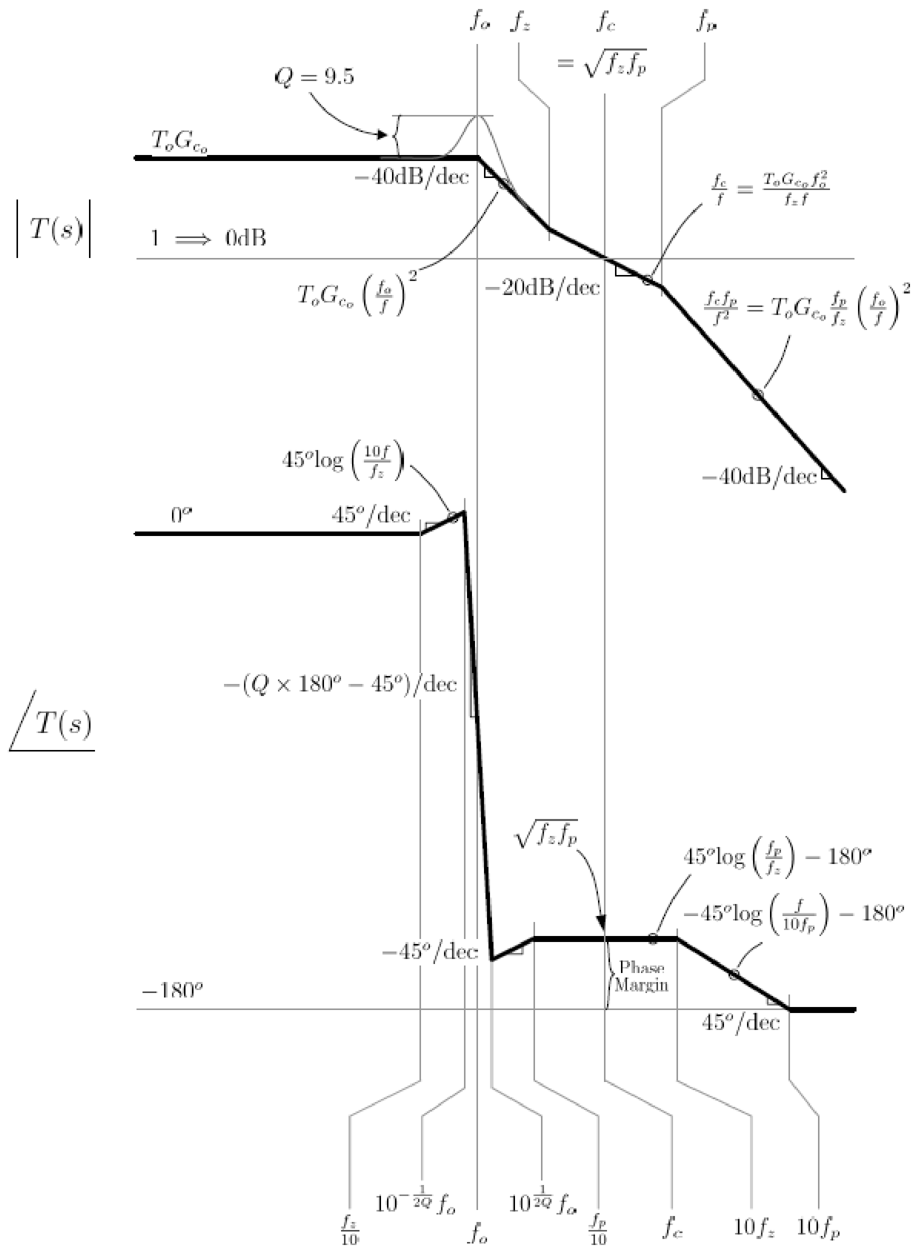
$$T_c(s) = G_c \cdot T(s)$$

$$\Longrightarrow$$

$$T_c(s) = G_{c_o} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \cdot \frac{T_o}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

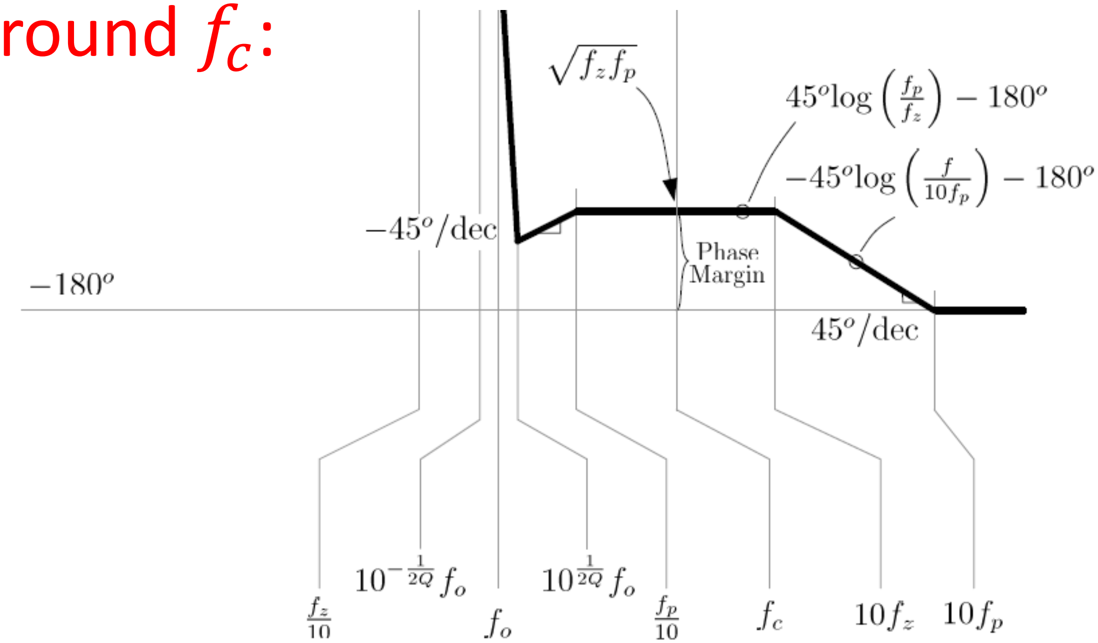
Lead Compensation

Combining:



Lead Compensation

Focusing around f_c:



$\Longrightarrow$ Phase margin: $PM = 45^\circ \log\left(\frac{f_p}{f_z}\right)$

For a desired phase margin of 45° :

$$45^\circ = 45^\circ \log\left(\frac{f_p}{f_z}\right)$$

$$\Longrightarrow \quad f_p = 10 f_z$$

Lead Compensation

Set the unity gain frequency, f_c: $\quad$ (e.g. let $f_c = 5$ kHz)

$$f_c = \sqrt{f_z f_p}$$

$$5 \text{ kHz} = \sqrt{10 {f_z}^2}$$

$$f_z = \frac{5 \text{ kHz}}{\sqrt{10}}$$

$$f_z = 1.58 \text{ kHz and } f_p = 15.8 \text{ kHz}$$

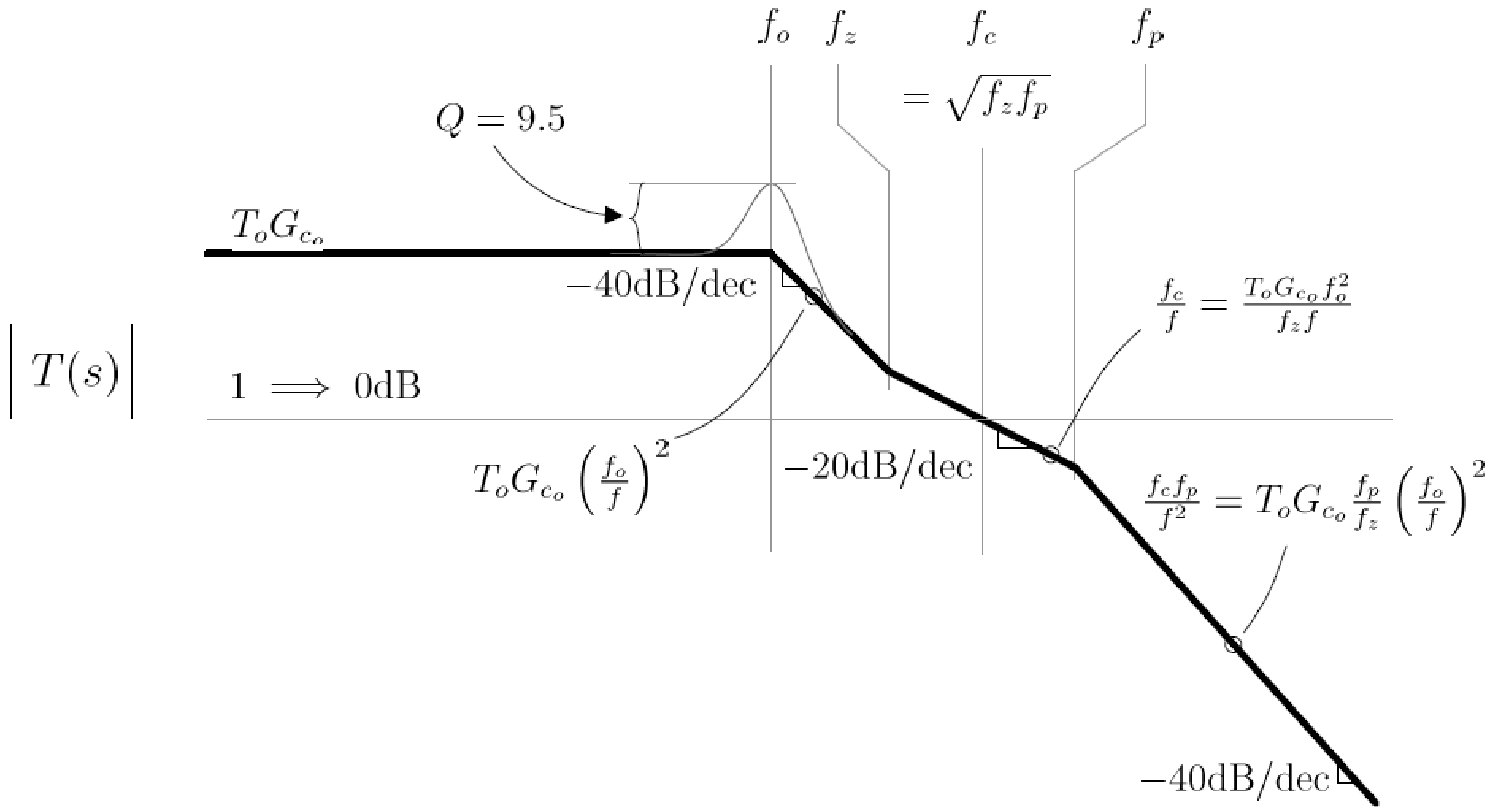
$$\Longrightarrow \qquad G_c(s) = G_{c_o} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}, \quad \omega_z < \omega_p$$

Found ω_z and ω_p, will next find G_{c_o}

Lead Compensation

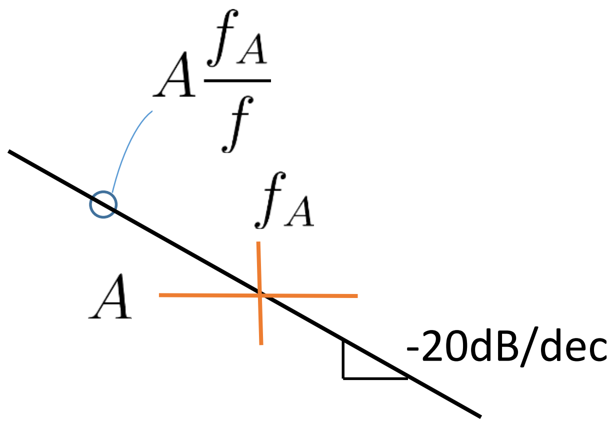
Finding G_{C_o}:

- Focus on magnitude response



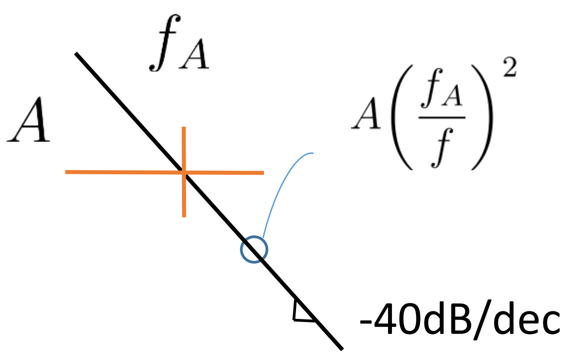
Lead Compensation

Bode asymptotic magnitude response review:



Given a magnitude of A at a frequency f_A, the magnitude expression for a line sloping at:

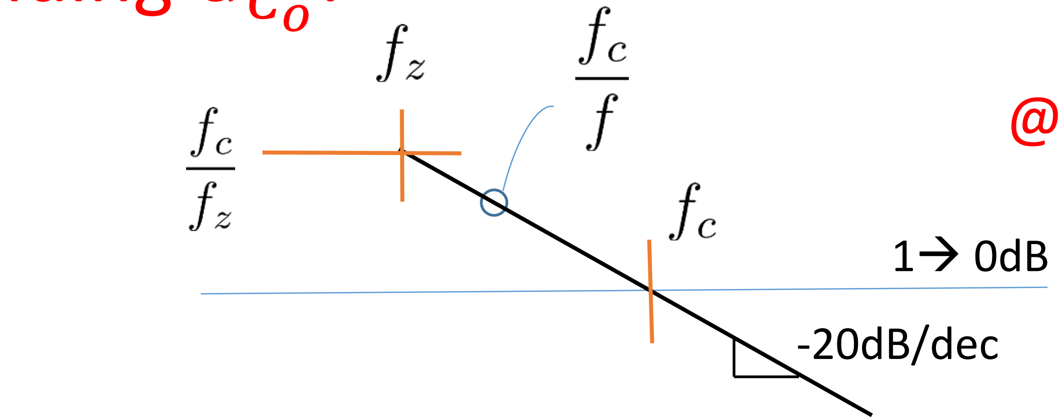
1) -20 dB/dec: $A\frac{f_A}{f}$



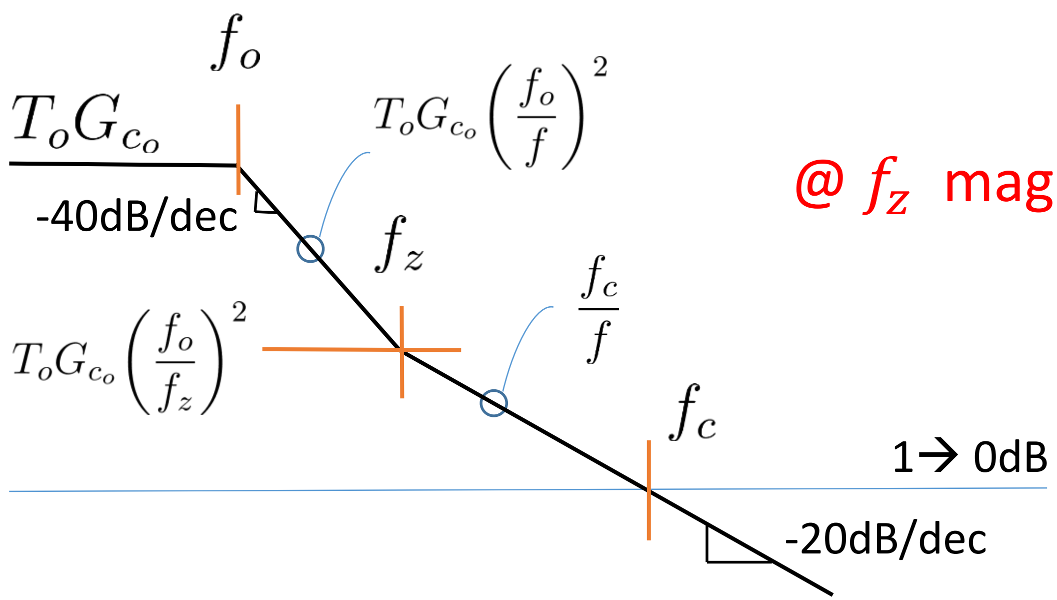
2) -40 dB/dec: $A\left(\frac{f_A}{f}\right)^2$

Lead Compensation

Finding G_{C_o}:



@ f_z magnitude is: $\frac{f_c}{f_z}$



@ f_z magnitude is: $\mathrm{T_o G_{c_o}} \left(\frac{f_o}{f_z}\right)^2$

Lead Compensation

Finding G_{C_o}:

- Equating the two expressions for the magnitude at f_z :

At f_z:

$$T_o G_{c_o} \left(\frac{f_o}{f_z}\right)^2 = \frac{f_c}{f_z}$$

$$\Longrightarrow \qquad G_{c_o} = \frac{1}{T_o}\left(\frac{f_z}{f_o}\right)^2 \frac{f_c}{f_z}$$

- All quantities on the right are known, so we can solve for G_{C_o}

Lead Compensation

Finding G_{c_o}:

$$G_{c_o} = \frac{1}{T_0}\left(\frac{f_z}{f_0}\right)^2 \frac{f_c}{f_z}$$

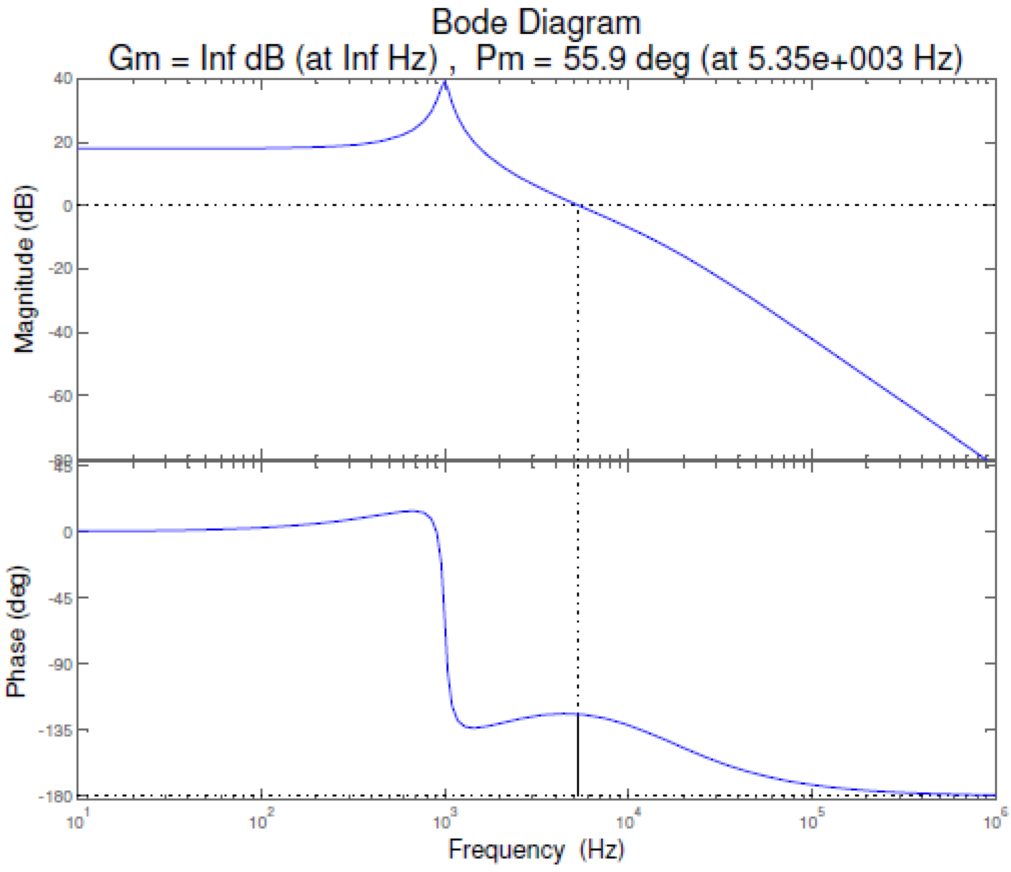
$$\Longrightarrow \qquad G_{c_o} = \frac{1}{2.33}\left(\frac{1.58\ \text{kHz}}{1\ \text{kHz}}\right)^2 \frac{5\ \text{kHz}}{1.58\ \text{kHz}}$$

$$\Longrightarrow \qquad G_{c_o} = 3.4$$

$\Longrightarrow$ Design of lead compensator is complete

Lead Compensation

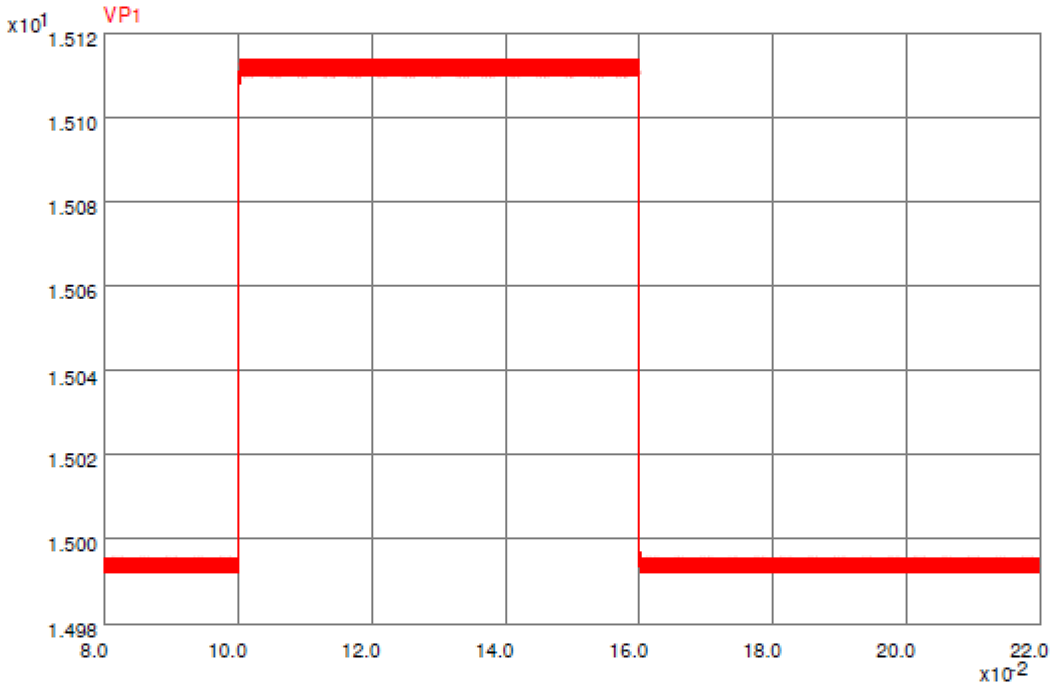
Exact loop gain using Matlab :



Phase margin $= 55.9°$

Lead Compensation

Time response to input voltage change using PECS:



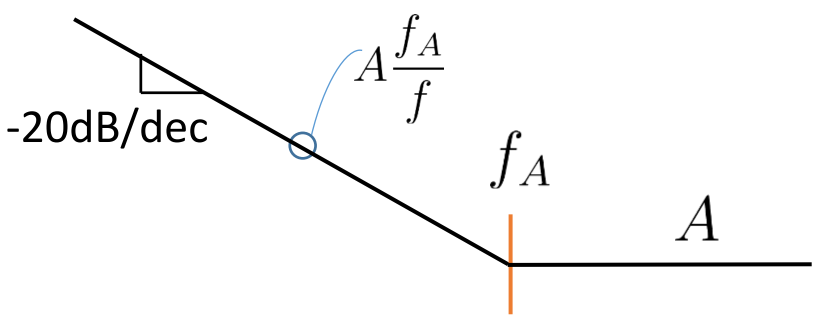
Non-zero steady state error

$\Longrightarrow$ Need a different compensator to null SS error

New Compensator:

Dominant Pole with Lead Compensation

Given the following magnitude response, what is the transfer function?:



Answer:

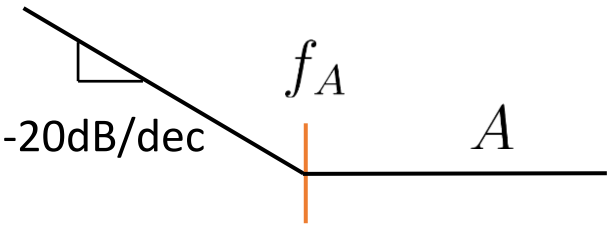
1) The low frequency asymptote is that of a pole at zero where the magnitude at f_A is A → $A\frac{\omega_A}{s}$

2) This is followed by a zero at f_A: $1+\frac{s}{\omega_A}$

3) Combining results in transfer function: $A\frac{\omega_A}{s}\left(1+\frac{s}{\omega_A}\right)$

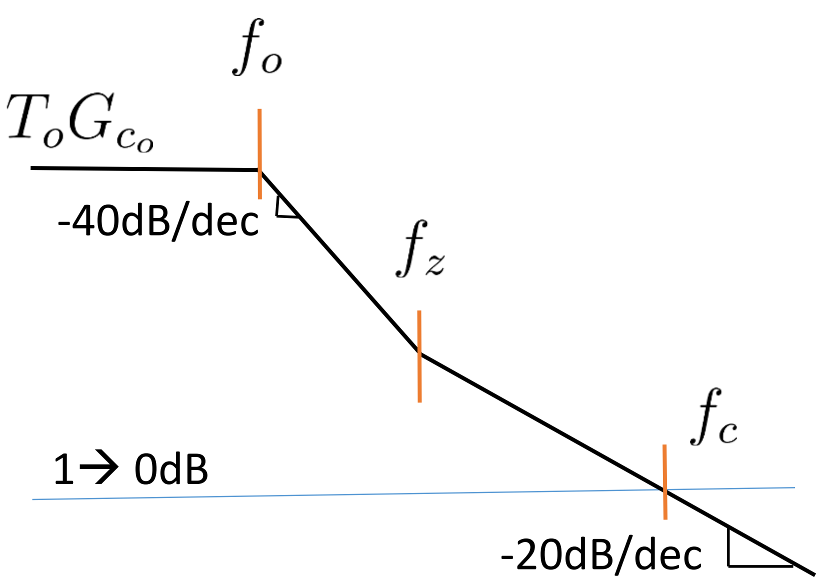
Dominant Pole with Lead Compensation

Added low frequency compensation:



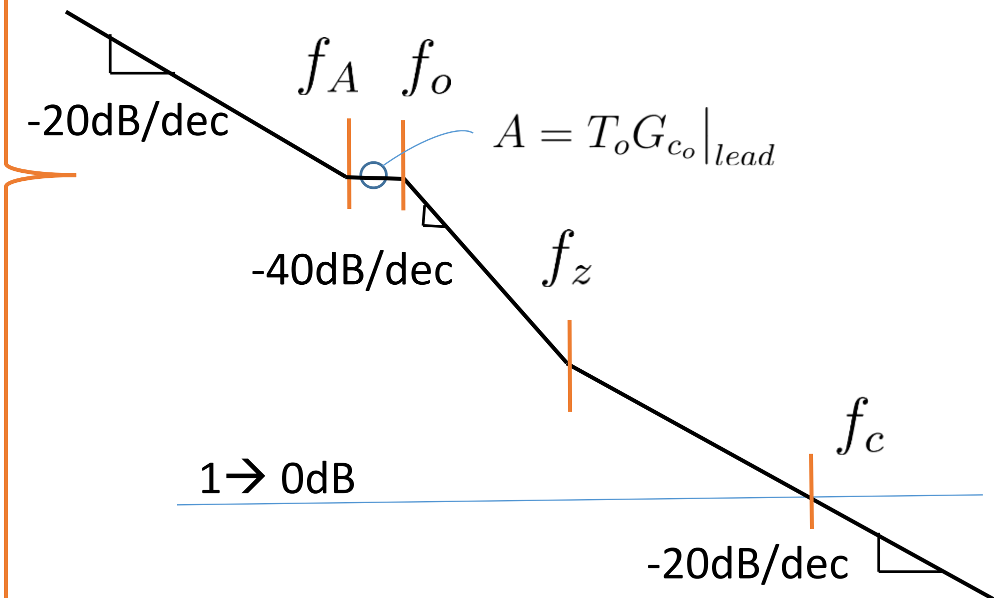
+

Lead compensated system:



Combining:

1) Set f_A (where $f_A \leq f_o$)
2) Adjust A to match low frequency magnitude $T_o G_{c_o}|_{\text{lead}}$



Dominant Pole with Lead Compensation

$$G_c(s) = \frac{\omega_I \left(1 + \frac{s}{\omega_A}\right)\left(1 + \frac{s}{\omega_z}\right)}{s\left(1 + \frac{s}{\omega_p}\right)}$$

where ω_I will next be determined

Compensated loop gain:

$$T_c(s) = G_c(s) \cdot T(s)$$

$$\Longrightarrow$$

$$T_c(s) = \frac{\omega_I \left(1 + \frac{s}{\omega_A}\right)\left(1 + \frac{s}{\omega_z}\right)}{s\left(1 + \frac{s}{\omega_p}\right)} \cdot \frac{T_o}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Dominant Pole with Lead Compensation

$$T_c(s) = \frac{\omega_I \left(1 + \frac{s}{\omega_A}\right)\left(1 + \frac{s}{\omega_z}\right)}{s\left(1 + \frac{s}{\omega_p}\right)} \cdot \frac{T_o}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

At frequency ω_A the magnitude of the compensated loop gain is given by:

$$\left|T_c(s)\right|_{s=j\omega_A} = \frac{\omega_I}{\omega_A} \cdot T_o \qquad (\omega_A < \omega_z, \omega_p, \omega_0)$$

This should equal the low frequency gain of the lead compensated loop gain:

$$\left|T_c(s)\right|_{s=j\omega_A} = \frac{\omega_I}{\omega_A} \cdot T_o = T_o G_{c_o}\big|_{lead}$$

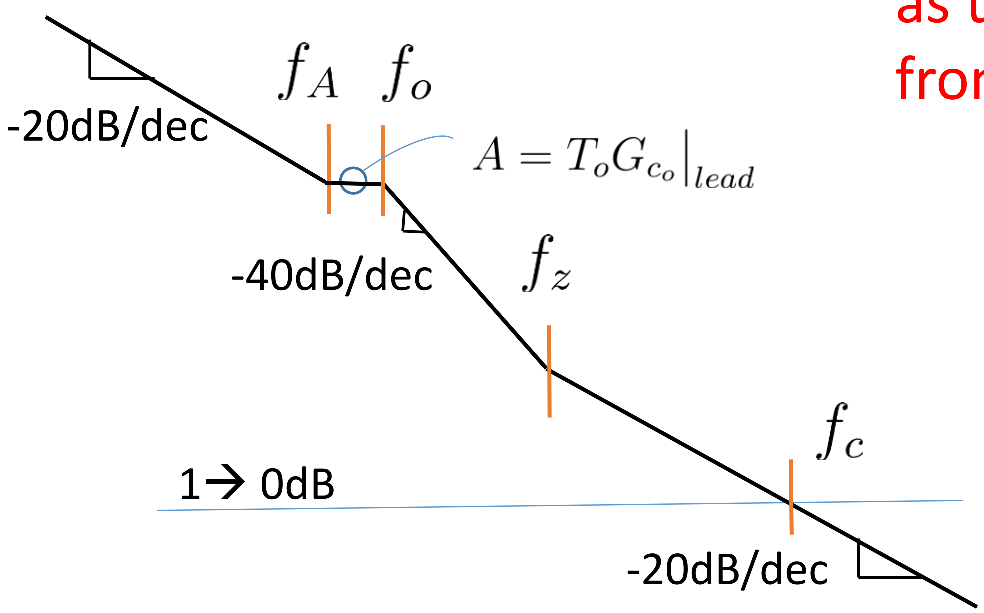
$$\Longrightarrow \qquad \omega_I = \omega_A \cdot G_{c_o}\big|_{lead}$$

Dominant Pole with Lead Compensation

$$\omega_I = \omega_A \cdot G_{c_o}\Big|_{lead}$$

$\omega_A = 2\pi f_A$. How to choose f_A?

Answer: $f_A \leq f_0$ and $f_A \leq \frac{f_c}{10}$ so as to maximize phase lead at f_c from the zero at f_A.



Note: There are disadvantages in making f_A too low (as shown in the next few slides)

Dominant Pole with Lead Compensation

Compensator transfer function:

$$G_c(s) = \frac{\omega_I \left(1 + \frac{s}{\omega_A}\right)\left(1 + \frac{s}{\omega_z}\right)}{s\left(1 + \frac{s}{\omega_p}\right)}$$

Four parameters needed to be determined:
$\omega_z, \omega_p, \omega_A$, and ω_I

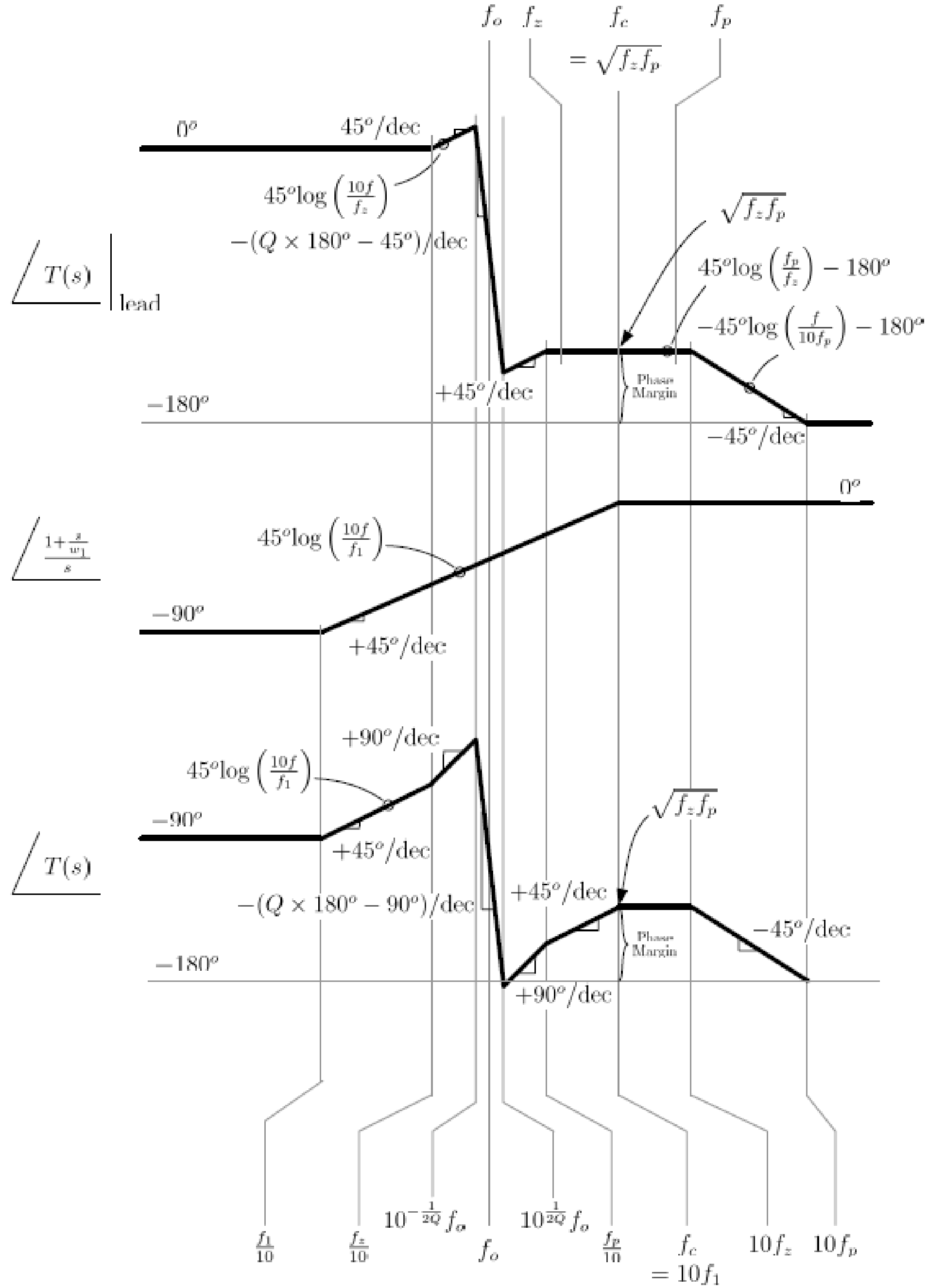
$\Longrightarrow$ Design of Dominant Pole with Lead Compensator is now complete

- Let's look closer at the how to choose ω_A (which also changes the value of ω_I)

Dominant Pole with Lead Compensation

Phase response of lead compensated loop gain combined with low frequency compensation with $f_A = \frac{f_C}{10}$:

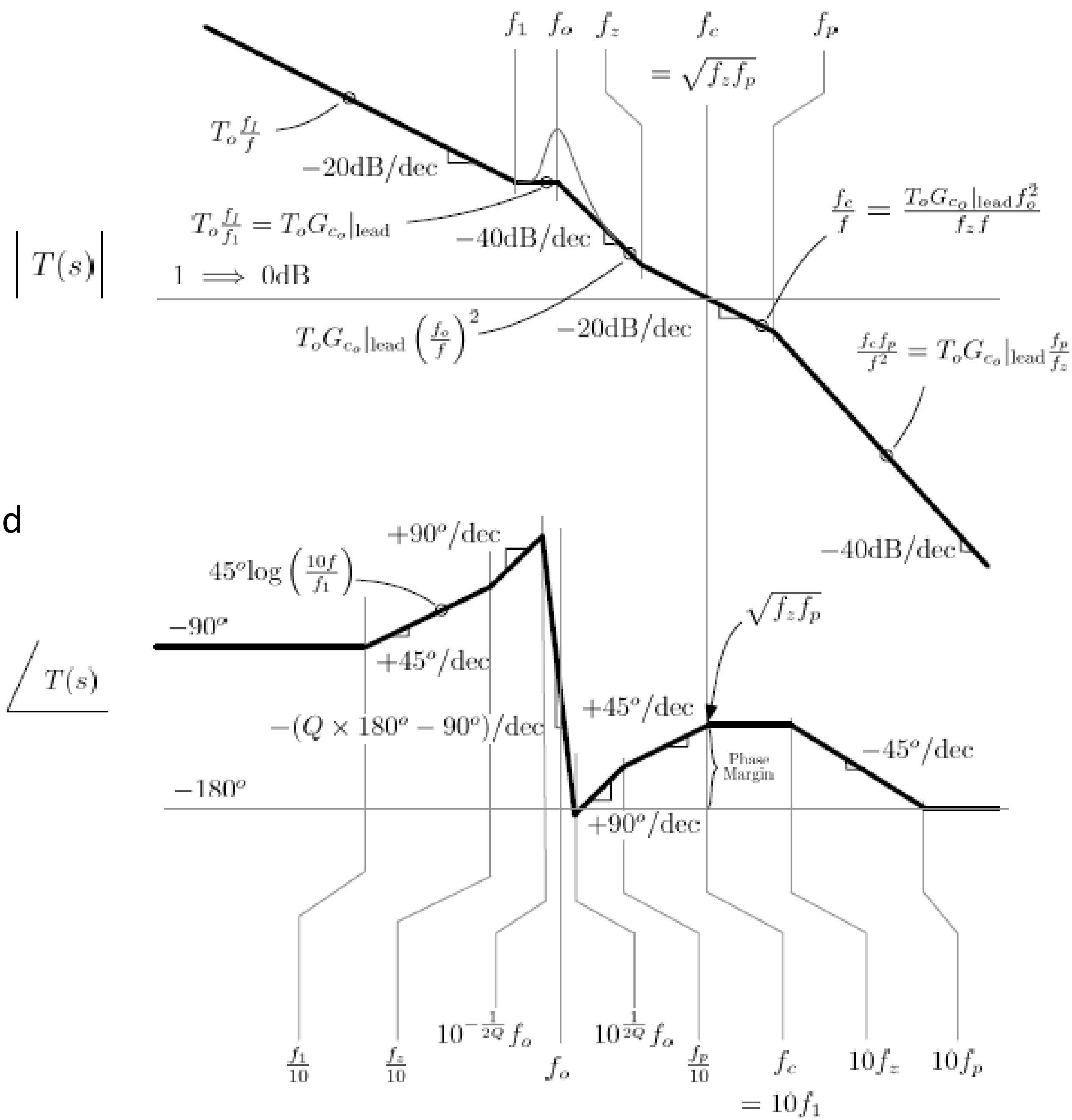
Note: f_A is denoted as f_1 in the figure along side (and in subsequent figures)



Dominant Pole with Lead Compensation

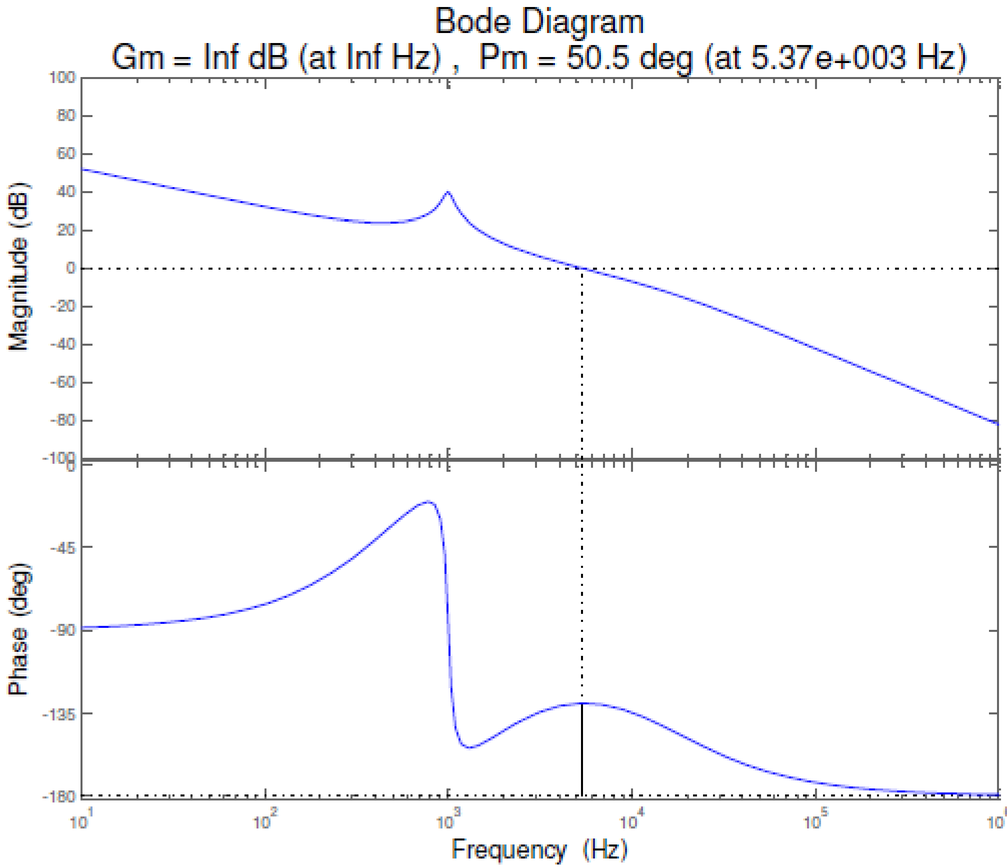
Asymptotic Bode plot for compensated loop gain $(f_A = \frac{f_c}{10})$:

Note: f_A is denoted as f_1 in the figure along side



Dominant Pole with Lead Compensation

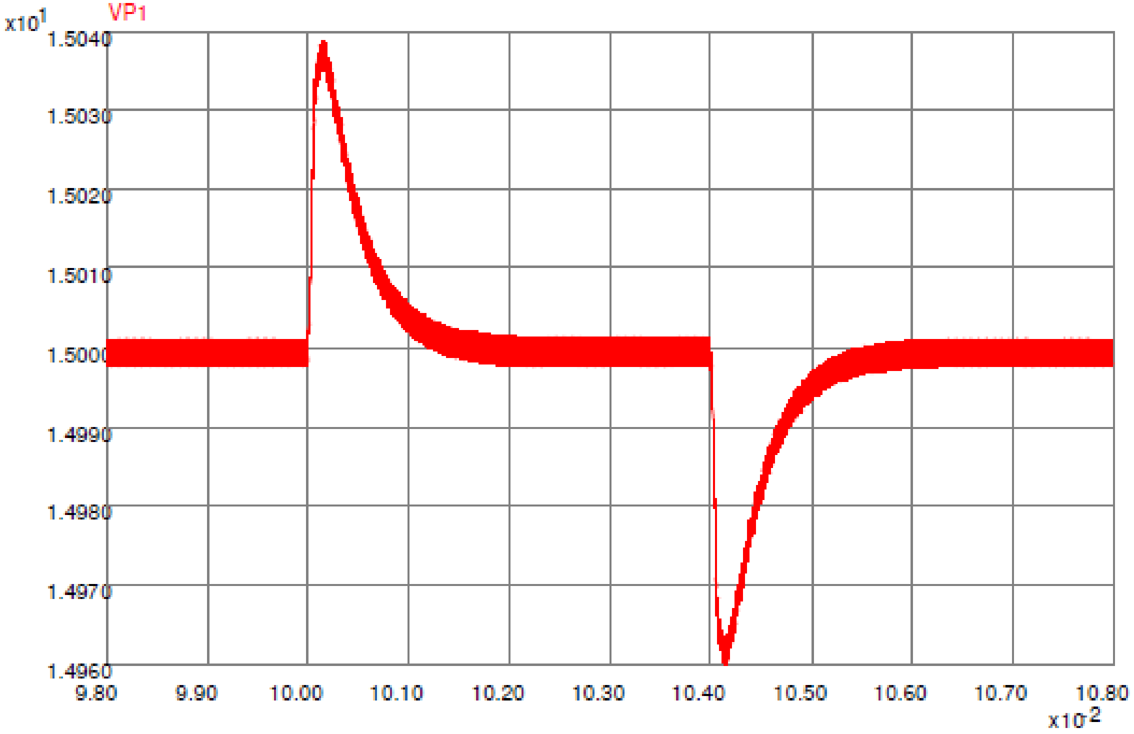
Exact Bode plot for compensated loop gain ($f_A = \frac{f_C}{10}$) :



Phase margin = $50.5°$

Dominant Pole with Lead Compensation

Associated time response to input voltage change:

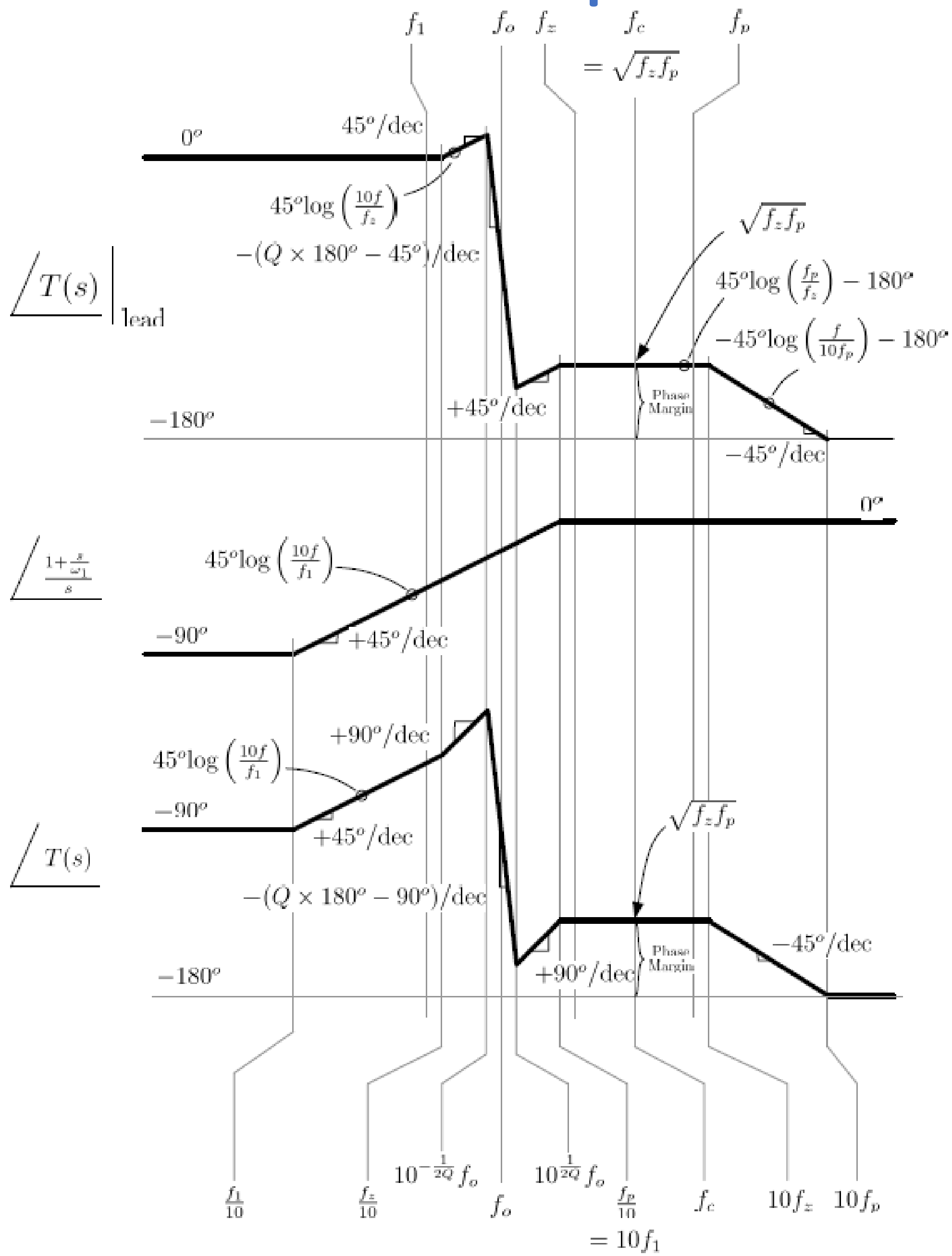


Settling time ≈ 1 ms

Dominant Pole with Lead Compensation

Phase response of lead compensated loop gain combined with low frequency compensation with $f_A = \frac{f_C}{33}$:

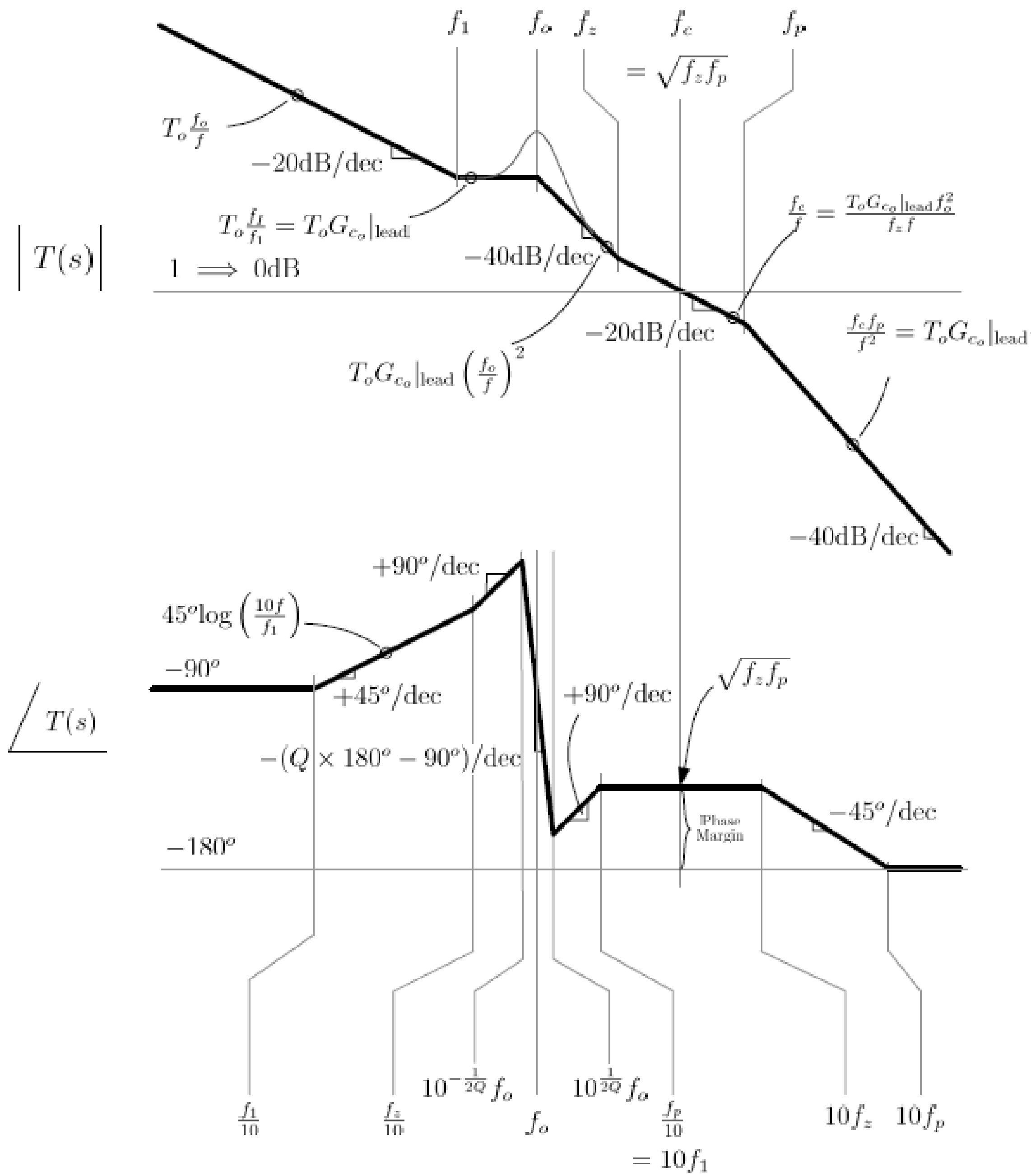
Note: f_A is denoted as f_1 in the figure along side



Dominant Pole with Lead Compensation

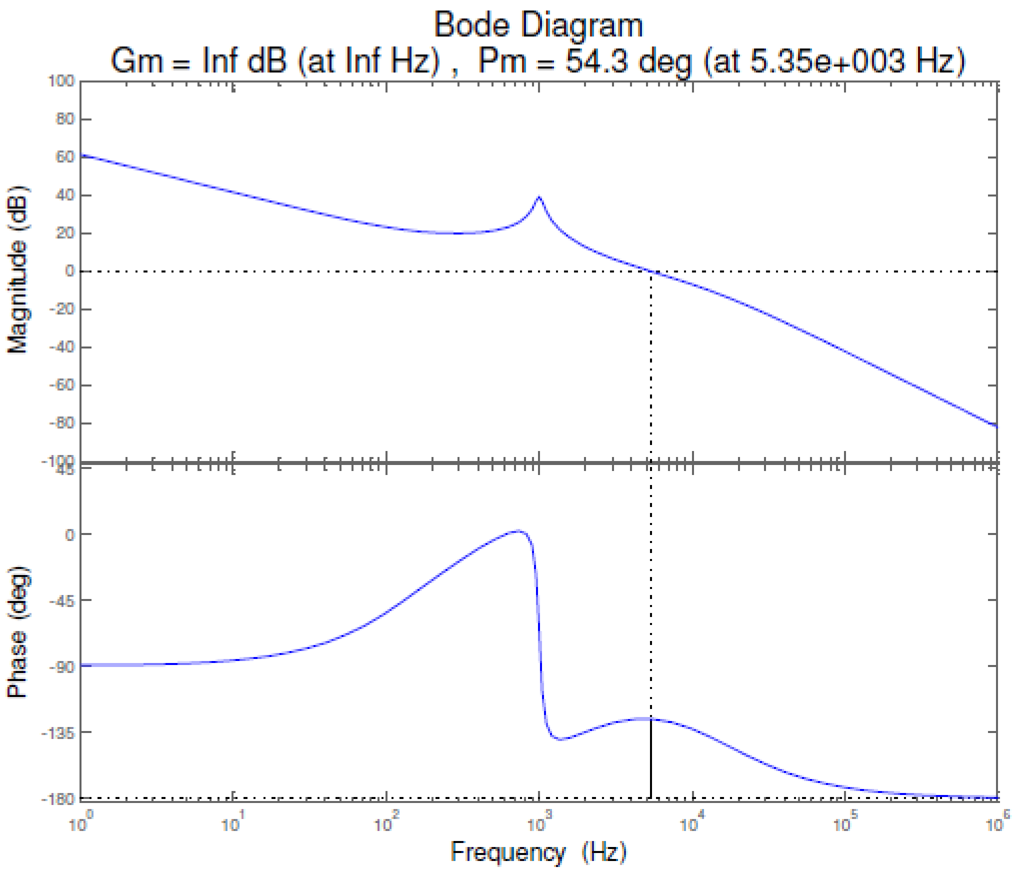
Asymptotic Bode plot for compensated loop gain $\left(f_A = \frac{f_c}{33}\right)$:

Note: f_A is denoted as f_1 in the figure along side



Dominant Pole with Lead Compensation

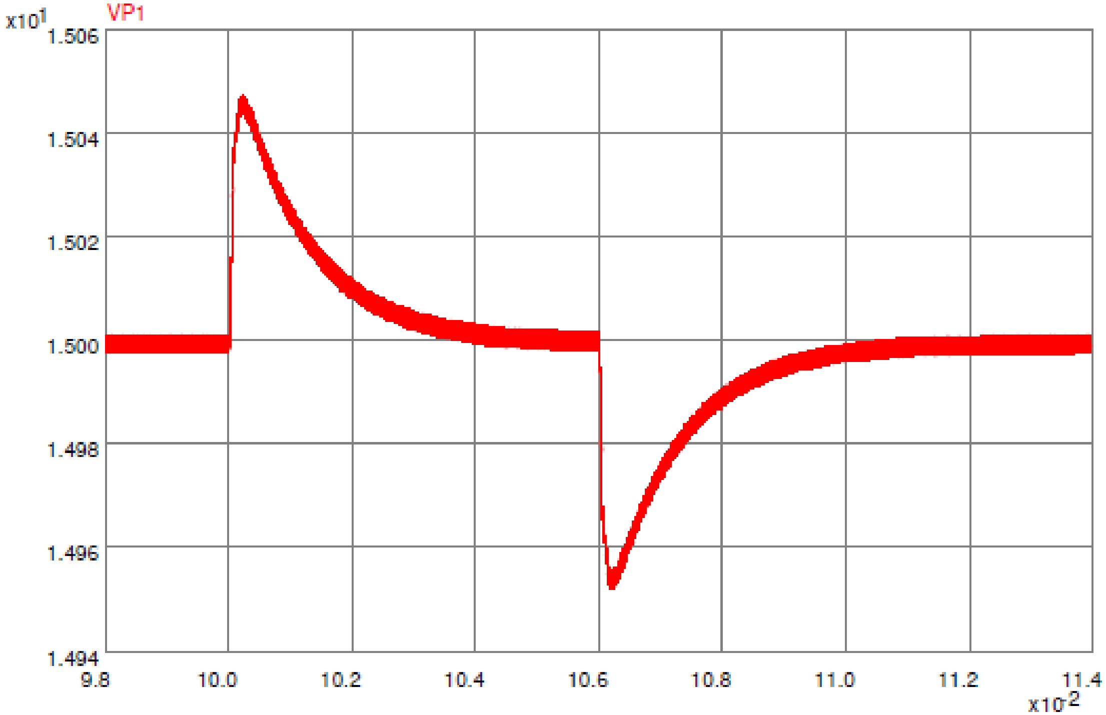
Exact Bode plot for compensated loop gain ($f_A = \frac{f_C}{33}$) :



Phase margin = $54.3°$

Dominant Pole with Lead Compensation

Associated time response to input voltage change:



Settling time ≈ 4 ms

⟹ Slower settling time

⟹ The first design is better

Summary

- Looked closely at the design of two compensators

  i) lead

  ii) dominant pole (integrator) with lead
- Derived the formulas needed to design, not just used available formulas
- Lead compensator: extends bandwidth while boosting the phase which can result in quick response with a good phase margin (minimal overshoot)
- dominant pole with lead compensator: has the properties of the lead compensator together with an integrator which provides zero steady state error
- Next, frequency domain specifications